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The standard model on non-commutative space-time: strong interactions included

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Abstract. This paper is a direct extension of our earlier work on electroweak currents and the Higgs sector in the standard model on non-commutative space-time, now with strong interactions included. Apart from the non-commutative corrections to standard model strong interactions, several new interactions appear. The most interesting ones are gluonic interactions with the electroweak sector. They are elaborated here in detail and the Feynman rules for interactions up to $\mathcal{O}(g_s^2\theta)$ are provided.

1 Introduction

This paper closely follows the paper by Melić et al. [1], extending it by including strong interactions. In this paper we present a careful discussion of QCD and QCD-electroweak charged and neutral currents in the non-commutative standard model (NCSM) [2] and compute the corresponding Yukawa parts of the action.

All relevant expressions are given in terms of physical fields and selected Feynman rules are provided with the aim to make the model more accessible to phenomenological considerations.

In Sect. 2 we briefly review the NCSM. The non-commutative QCD and QCD interactions with the electroweak and Yukawa sector are presented in Sects. 3 and 4. The selected Feynman rules to $\mathcal{O}(g_s^2\theta)$ are summarized in Sect. 5, while Sect. 6 is devoted to concluding remarks.

2 Non-commutative standard model

The action of the non-commutative standard model (NCSM) formally resembles the action of the classical standard model (SM): the usual point-wise products in the Lagrangian are replaced by the Moyal–Weyl product and (matter and gauge) fields are replaced by non-commutative fermion and gauge fields (denoted by a hat) which are expressed (via Seiberg–Witten maps) in terms of ordinary fermion and gauge fields ψ and V_μ:

$$\begin{aligned}\widehat{\Psi} &= \widehat{\Psi}[V] \qquad (1)\\ &= \psi - \frac{1}{2}\,\theta^{\alpha\beta}\,V_\alpha\,\partial_\beta\psi + \frac{\mathrm{i}}{8}\,\theta^{\alpha\beta}\,[V_\alpha, V_\beta]\,\psi + \mathcal{O}(\theta^2),\\ \widehat{V}_\mu &= \widehat{V}_\mu[V]\\ &= V_\mu + \frac{1}{4}\theta^{\alpha\beta}\{\partial_\alpha V_\mu + F_{\alpha\mu}, V_\beta\} + \mathcal{O}(\theta^2). \qquad (2)\end{aligned}$$

Here, $F_{\mu\nu}$ is the ordinary field strength and V_μ is the gauge potential corresponding to the SM gauge group structure

$$V_\mu(x) = g'\mathcal{A}_\mu(x)Y + g\sum_{a=1}^{3} B_\mu^a(x)\,T_{\mathrm{L}}^a + g_s\sum_{b=1}^{8} G_\mu^b(x)\,T_S^b, \qquad (3)$$

and $T_{\mathrm{L}}^a = \tau_a/2$, $T_S^b = \lambda^b/2$, with τ^a and λ^b being the Pauli and Gell-Mann matrices, respectively. Here, $\mathcal{A}_\mu$, B_μ^a and G_μ^b represent ordinary $U(1)_Y$, $SU(2)_{\mathrm{L}}$ and $SU(3)_C$ fields, respectively.

The approach to non-commutative field theory based on star products and Seiberg–Witten maps allows the generalization of the SM of particle physics to the case of non-commutative space-time, keeping the original gauge group and particle content [2–11]. It provides a systematic way to compute Lorentz violating operators that could be a signature of a (hypothetical) non-commutative space-time structure [12–22]. In this paper we do not repeat the derivation, but take the NCSM action $S_{\mathrm{NCSM}} = S_{\mathrm{fermions}} + S_{\mathrm{gauge}} + S_{\mathrm{Higgs}} + S_{\mathrm{Yukawa}}$, given already in terms of commutative SM fields, as a starting point. Note that the S_{NCSM} action is anomaly free [23].

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With respect to the gauge sector there are two models: the minimal non-commutative standard model (mNCSM) [2] and the non-minimal non-commutative standard model (nmNCSM) [16]. The main difference between the two models is due to the freedom of the choice of traces in the kinetic terms for gauge fields. In the mNCSM, we adopt the representation that yields a model as close as possible to the SM without new triple-gauge boson couplings. In the nmNCSM [16] the trace is chosen over all particles on which covariant derivatives act and which have different quantum numbers. In the SM, these are five multiplets for each generation of fermions and one Higgs multiplet.

Note that the fermion sector of the action S_{NCSM} is not affected by choosing different traces over the representations in the gauge part of the action and remains the same in both models.

2.1 Minimal/non-minimal NCSM

The minimal NCSM gauge action is given by [1]

$$
\begin{aligned}
&S^{\mathrm{mNCSM}}_{\mathrm{gauge}}\\
&= -\frac{1}{2}\int \mathrm{d}^4x \left(\frac{1}{2}\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} + \mathrm{Tr}\, B_{\mu\nu}B^{\mu\nu} + \mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}\right)\\
&\quad + \frac{1}{4}\, g_s\, d^{abc}\, \theta^{\rho\sigma} \int \mathrm{d}^4x \left(\frac{1}{4} G^a_{\rho\sigma} G^b_{\mu\nu} - G^a_{\rho\mu} G^b_{\sigma\nu}\right) G^{\mu\nu,c}\\
&\quad + \mathcal{O}(\theta^2),
\end{aligned}
\tag{4}
$$

where $\mathcal{A}_{\mu\nu}$, $B_{\mu\nu}(= B^a_{\mu\nu}T^a_{\mathrm{L}})$ and $G_{\mu\nu}(= G^a_{\mu\nu}T^a_S)$ denote field strengths.

In the non-minimal NCSM, the gauge action (4) is extended by new gauge terms, (27) in [1], from where the Lagrangians including gluons are derived

$$
\begin{aligned}
\mathcal{L}_{\gamma gg} &= \frac{e}{4}\sin 2\theta_{\mathrm{W}}\, \mathrm{K}_{\gamma gg}\, \theta^{\rho\sigma} \left[2A^{\mu\nu}\left(2G^a_{\mu\rho}G^b_{\nu\sigma} - G^a_{\mu\nu}G^b_{\rho\sigma}\right)\right.\\
&\quad \left. + 8A_{\mu\rho}G^{\mu\nu,a}G^b_{\nu\sigma} - A_{\rho\sigma}G^a_{\mu\nu}G^{\mu\nu,b}\right]\delta^{ab}, \qquad (5)\\
\mathcal{L}_{Zgg} &= \mathcal{L}_{\gamma gg}(A_\mu \to Z_\mu), \qquad (6)
\end{aligned}
$$

with the following coupling constants:

$$
\mathrm{K}_{\gamma gg} = \frac{-g_s^2}{2gg'}\left(g'^2 + g^2\right)\kappa_3, \quad \mathrm{K}_{Zgg} = -\frac{g'}{g}\,\mathrm{K}_{\gamma gg}. \tag{7}
$$

Details of the derivations of neutral triple-gauge boson terms and the properties of the coupling constants in (7) are given in [16, 17].

3 QCD and QCD-electroweak matter currents

The general fermionic action in NCSM reads [1]

$$
\begin{aligned}
S_\psi &= \int \mathrm{d}^4x \left(\mathrm{i}\overline{\psi}\, \not{D}\, \psi - \frac{\mathrm{i}}{4}\, \overline{\psi}\, \theta^{\mu\nu\rho}\, \mathcal{R}_\psi(F_{\mu\nu})\, D_\rho\psi\right)\\
&\quad + \mathcal{O}(\theta^2),
\end{aligned}
\tag{8}
$$

where $\theta^{\mu\nu\rho} = \theta^{\mu\nu}\gamma^\rho + \theta^{\nu\rho}\gamma^\mu + \theta^{\rho\mu}\gamma^\nu$ is a totally antisymmetric quantity, ψ denotes any fermion field, while the corresponding representations $\mathcal{R}_\psi$ for various fermion fields are listed in Table 2 of [1].

We express the NCSM currents in terms of physical fields starting with the left-handed sector. In (8), the representation $\mathcal{R}_{\Psi_{\mathrm{L}}}(V_\mu)$ of the SM gauge group takes the form

$$
\mathcal{R}_{\Psi_{\mathrm{L}}}(V_\mu) = g'\, \mathcal{A}_\mu\, Y_{\Psi_{\mathrm{L}}} + g\, B^a_\mu T^a_{\mathrm{L}} + g_s\, G^b_\mu T^b_S. \tag{9}
$$

The hypercharge generator $Y_{\Psi_{\mathrm{L}}}$ can be rewritten as $Y_{\Psi_{\mathrm{L}}} = Q_{q_{\mathrm{up}}} - T_{3,q_{\mathrm{up,L}}} = Q_{q_{\mathrm{down}}} - T_{3,q_{\mathrm{down,L}}}$. Then the left-handed electroweak part of the action S_ψ can be cast in the form

$$
\begin{aligned}
S_{\psi,ew,\mathrm{L}} &= \int \mathrm{d}^4x \left(\bar{\Psi}_{\mathrm{L}}\, \mathrm{i}\not{\partial}\, \Psi_{\mathrm{L}} + \bar{\Psi}_{\mathrm{L}}\, \mathbf{J}^{(\mathrm{L})}\, \Psi_{\mathrm{L}}\right),\\
\mathbf{J}^{(\mathrm{L})} &= \bar{q}_{\mathrm{up,L}}\, J^{(\mathrm{L})}_{12}\, q_{\mathrm{down,L}} + \bar{q}_{\mathrm{down,L}}\, J^{(\mathrm{L})}_{21}\, q_{\mathrm{up,L}} \qquad (10)\\
&\quad + \bar{q}_{\mathrm{up,L}}\, J^{(\mathrm{L})}_{11}\, q_{\mathrm{up,L}} + \bar{q}_{\mathrm{down,L}}\, J^{(\mathrm{L})}_{22}\, q_{\mathrm{down,L}},
\end{aligned}
$$

where $\mathbf{J}^{(\mathrm{L})}$ is a 2×2 matrix whose off-diagonal elements ($J^{(\mathrm{L})}_{12}$, $J^{(\mathrm{L})}_{21}$) denote the charged currents and diagonal elements ($J^{(\mathrm{L})}_{11}$, $J^{(\mathrm{L})}_{22}$) the neutral currents. After some algebra, with gluons included ($G_\mu = G^a_\mu T^a_S$), we obtain

$$
\begin{aligned}
J^{(\mathrm{L})}_{12} &= \frac{g}{\sqrt{2}}\not{W}^+ + J^{(\mathrm{L},\theta)}_{12} + \mathcal{O}(\theta^2), \qquad (11a)\\
J^{(\mathrm{L})}_{21} &= \frac{g}{\sqrt{2}}\not{W}^- + J^{(\mathrm{L},\theta)}_{21} + \mathcal{O}(\theta^2), \qquad (11b)
\end{aligned}
$$

$$
\begin{aligned}
J^{(\mathrm{L})}_{11} &= \left[e\, Q_{q_{\mathrm{up}}}\, \not{A}\right.\\
&\quad \left. + \frac{g}{\cos\theta_{\mathrm{W}}}(T_{3,q_{\mathrm{up,L}}} - Q_{q_{\mathrm{up}}}\, \sin^2\theta_{\mathrm{W}})\not{Z} + g_s\not{G}\right]\\
&\quad + J^{(\mathrm{L},\theta)}_{11} + \mathcal{O}(\theta^2), \qquad (12a)\\
J^{(\mathrm{L})}_{22} &= \left[e\, Q_{q_{\mathrm{down}}}\, \not{A}\right.\\
&\quad \left. + \frac{g}{\cos\theta_{\mathrm{W}}}(T_{3,q_{\mathrm{down,L}}} - Q_{q_{\mathrm{down}}}\, \sin^2\theta_{\mathrm{W}})\not{Z} + g_s\not{G}\right]\\
&\quad + J^{(\mathrm{L},\theta)}_{22} + \mathcal{O}(\theta^2), \qquad (12b)
\end{aligned}
$$

where $J^{(\mathrm{L},\theta)}_{12}$ and $J^{(\mathrm{L},\theta)}_{11}$, (41)–(43) in [1], receive the following additional contributions from the inclusion of gluons:

$$
\begin{aligned}
&J^{(\mathrm{L},\theta)}_{12}:\\
&\frac{g}{2\sqrt{2}}\,\theta^{\mu\nu\rho}\, W^+_\mu \left\{g_s\left[G_\nu\, (\overleftarrow{\partial}_\rho + \overrightarrow{\partial}_\rho) + 2(\partial_\rho G_\nu)\right]\right.\\
&-\mathrm{i}\, e\, g_s(Q_{q_{\mathrm{up}}} - Q_{q_{\mathrm{down}}})A_\nu G_\rho\\
&-\mathrm{i}\, g_s\, \frac{g}{\cos\theta_{\mathrm{W}}}\\
&\times\Big((T_{3,q_{\mathrm{up,L}}} - T_{3,q_{\mathrm{down,L}}})\\
&\qquad -(Q_{q_{\mathrm{up}}} - Q_{q_{\mathrm{down}}})\, \sin^2\theta_{\mathrm{W}}\Big)\, Z_\nu G_\rho\\
&\left. +\mathrm{i}\, g_s^2\, G_\nu G_\rho\right\},
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&J_{11}^{(\mathrm{L},\theta)}:\\
&\frac{1}{2}\,\theta^{\mu\nu\rho}\Big\{\mathrm{i}\,g_s\,(\partial_\nu G_\mu)\;\overrightarrow{\partial}_\rho\\
&-g_s^2\,\Big[G_\mu G_\nu\;\overrightarrow{\partial}_\rho+(\partial_\rho G_\mu)G_\nu\Big]+\mathrm{i}g_s^3\,G_\mu G_\nu G_\rho\\
&-e\,g_s\,Q_{q_\mathrm{up}}\;[(\partial_\rho A_\mu)\,G_\nu-A_\mu\,(\partial_\rho G_\nu)]\\
&-\frac{g}{\cos\theta_\mathrm{W}}\,g_s(T_{3,q_\mathrm{up,L}}-Q_{q_\mathrm{up}}\;\sin^2\theta_\mathrm{W})\\
&\times[(\partial_\rho Z_\mu)\,G_\nu-Z_\mu(\partial_\rho\,G_\nu)]\\
&+\mathrm{i}\,\frac{g^2}{2}\,g_s\;W_\mu^+\,W_\nu^-\,G_\rho+\mathrm{i}\,e\,Q_{q_\mathrm{up}}\,g_s^2\,A_\mu\,G_\nu\,G_\rho \qquad (14)\\
&+\mathrm{i}\,\frac{g}{\cos\theta_\mathrm{W}}\,g_s^2(T_{3,q_\mathrm{up,L}}-Q_{q_\mathrm{up}}\;\sin^2\theta_\mathrm{W})\,Z_\mu\,G_\nu\,G_\rho\Big\},
\end{aligned}
$$

while

$$
\left.\begin{matrix}J_{21}^{(\mathrm{L},\theta)}\\ J_{22}^{(\mathrm{L},\theta)}\end{matrix}\right\}\to\left\{\begin{matrix}J_{12}^{(\mathrm{L},\theta)}\\ J_{11}^{(\mathrm{L},\theta)}\end{matrix}\right. \qquad (15)
$$

under $(W^+\leftrightarrow W^-, Q_{q_\mathrm{up}}\leftrightarrow Q_{q_\mathrm{down}}, T_{3,q_\mathrm{up,L}}\leftrightarrow T_{3,q_\mathrm{down,L}})$. In this paper we use the notation $(\partial_\rho q\equiv\overrightarrow{\partial}_\rho\,q)$ and $(\partial_\rho\bar{q}\equiv\bar{q}\,\overleftarrow{\partial}_\rho)$.

For the right-handed sector q_R represents $q_\mathrm{R}\in\{q_\mathrm{up,R},q_\mathrm{down,R}\}$, and the representation $\mathcal{R}_{q_\mathrm{R}}(V_\mu)$ is given by

$$
\mathcal{R}_{q_\mathrm{R}}(V_\mu)=g'\,\mathcal{A}_\mu+Y_{q_\mathrm{R}}\,g_s\,G_\mu, \qquad (16)
$$

with SU(3) fields included. Note that for the right-handed fermions we have $T_{3,q_\mathrm{R}}=0$ and $Y_{q_\mathrm{R}}=Q_q$. The right-handed electroweak part of the action S_ψ

$$
\begin{aligned}
S_{\psi,ew,\mathrm{R}}&=\int\mathrm{d}^4x\,\Big(\bar{q}_\mathrm{R}\,\mathrm{i}\partial\!\!\!/\,q_\mathrm{R}+\bar{q}_\mathrm{R}\,J^{(\mathrm{R})}\,q_\mathrm{R}\Big),\\
J^{(\mathrm{R})}&=[e\,Q_q\,A\!\!\!/-eQ_q\,\tan\theta_\mathrm{W}Z\!\!\!/+g_s\,G\!\!\!/]+J^{(\mathrm{R},\theta)}\\
&\quad+\mathcal{O}(\theta^2), \qquad (17)
\end{aligned}
$$

has the following additional gluon contributions to (48) in [1]:

$$
\begin{aligned}
J^{(\mathrm{R},\theta)}:\;&\frac{1}{2}\,\theta^{\mu\nu\rho}\Big\{\mathrm{i}\,g_s\,(\partial_\nu G_\mu)\;\overrightarrow{\partial}_\rho\\
&-g_s^2\,\Big[G_\mu G_\nu\;\overrightarrow{\partial}_\rho+(\partial_\rho G_\mu)G_\nu\Big]+\mathrm{i}g_s^3\,G_\mu G_\nu G_\rho\\
&-g_s\,e\,Q_q\;[(\partial_\rho A_\mu)\,G_\nu-A_\mu\,(\partial_\rho G_\nu)]\\
&+g_s\,e\,Q_q\;\tan\theta_\mathrm{W}\;[(\partial_\rho Z_\mu)\,G_\nu-Z_\mu(\partial_\rho\,G_\nu)]\\
&+\mathrm{i}\,g_s^2\,e\,Q_q\;[A_\mu-\tan\theta_\mathrm{W}Z_\mu]\;G_\nu\,G_\rho\Big\}. \qquad (18)
\end{aligned}
$$

The full electroweak part of the action $S_{\psi,ew}=S_{\psi,ew,\mathrm{L}}+S_{\psi,ew,\mathrm{R}}$ takes the form given in (49) from [1]. As explained there, the weak eigenstates (q_up, q_down) are transformed into the physical mass quark eigenstates ($u^{(i)}$, $d^{(i)}$).

4 Yukawa terms

By an analysis similar to the one in the preceeding section, one can show that the Yukawa part of the action,

$$
\begin{aligned}
S_{\psi,\mathrm{Yukawa}}=\int\mathrm{d}^4x\sum_{i,j=1}^{3}\Big[&\bar{d}^{(i)}\Big(N_{dd}^{V(ij)}+\gamma_5\,N_{dd}^{A(ij)}\Big)\,d^{(j)}\\
&+\bar{u}^{(i)}\Big(N_{uu}^{V(ij)}+\gamma_5\,N_{uu}^{A(ij)}\Big)\,u^{(j)}\\
&+\bar{u}^{(i)}\Big(C_{ud}^{V(ij)}+\gamma_5\,C_{ud}^{A(ij)}\Big)\,d^{(j)}\\
&+\bar{d}^{(i)}\Big(C_{du}^{V(ij)}+\gamma_5\,C_{du}^{A(ij)}\Big)\,u^{(j)}\Big], \qquad (19)
\end{aligned}
$$

contains additional gluon contributions to the neutral currents ((67)–(69) from [1]):

$$
\begin{aligned}
&N_{dd}^{V,\theta(ij)}:\\
&-\frac{1}{2}\,g_s\,\theta^{\mu\nu}M_\mathrm{down}^{(ij)} \qquad (20)\\
&\times\Big\{-G_\mu\,\frac{(\partial_\nu h)}{v}+[\partial_\nu\,G_\mu+\mathrm{i}g_s\,G_\mu G_\nu]\Big(1+\frac{h}{v}\Big)\Big\},\\
&N_{dd}^{A,\theta(ij)}:\\
&\frac{\mathrm{i}gg_s}{2\cos\theta_\mathrm{W}}\,\theta^{\mu\nu}M_\mathrm{down}^{(ij)}\,T_{3,\psi_\mathrm{down,L}}\,Z_\mu G_\nu\Big(1+\frac{h}{v}\Big), \qquad (21)
\end{aligned}
$$

as well as to the charge currents, (71) and (72) in [1],

$$
\begin{aligned}
&C_{ud}^{V,\theta(ij)}:\\
&-\frac{\mathrm{i}gg_s}{4\sqrt{2}}\theta^{\mu\nu}\Big(1+\frac{h}{v}\Big)\\
&\times\Big[(VM_\mathrm{down})^{(ij)}-(M_\mathrm{up}V)^{(ij)}\Big]\,G_\mu W_\nu^+,\\
&C_{ud}^{A,\theta(ij)}: \qquad (22)\\
&-\frac{\mathrm{i}gg_s}{4\sqrt{2}}\theta^{\mu\nu}\Big(1+\frac{h}{v}\Big)\\
&\times\Big[(VM_\mathrm{down})^{(ij)}+(M_\mathrm{up}V)^{(ij)}\Big]\,G_\mu W_\nu^+.
\end{aligned}
$$

In the above, $M_\mathrm{up,down}^{(ij)}$ are the usual 3×3 diagonal mass matrices defined in [1], and

$$
\begin{aligned}
&\left.\begin{matrix}N_{uu}^{V,\theta(ij)}\\ N_{uu}^{A,\theta(ij)}\end{matrix}\right\}=\left\{\begin{matrix}N_{dd}^{V,\theta(ij)}\\ N_{dd}^{A,\theta(ij)}\end{matrix}\right.\;(W^+\leftrightarrow W^-,\mathrm{down}\to\mathrm{up}),\\
&C_{du}^{V(ij)}=\Big(C_{ud}^{V(ij)}(\overrightarrow{\partial}\leftrightarrow\overleftarrow{\partial})\Big)^\dagger,\\
&C_{du}^{A(ij)}=-\Big(C_{ud}^{A(ij)}(\overrightarrow{\partial}\leftrightarrow\overleftarrow{\partial})\Big)^\dagger, \qquad (23)
\end{aligned}
$$

while $V^{(ij)}$ are the standard CKM matrix elements.

5 Feynman rules

In this section, we list a number of selected Feynman rules for the non-commutative QCD and non-commutative

QCD-electroweak sectors up to the first order in θ and up to g_s^2 order. Higher-order terms are not considered in this work.

The following notation for vertices has been adopted: all gauge boson lines are taken to be incoming; the momenta of the incoming and outgoing quarks, following the flow of the quark line are given by $p_{\rm in}$ and $p_{\rm out}$, respectively. In the following we denote quarks by $q \in \{u^{(i)}, d^{(i)}\}$, and the generation indices by i and j. In the Feynman rules we use the following definitions:

$$
\begin{aligned}
c_{V,q} &= T_{3,q_{\rm L}} - 2\,Q_q \sin^2\theta_{\rm W}, \\
c_{A,q} &= T_{3,q_{\rm L}}.
\end{aligned} \tag{24}
$$

We also make use of $(\theta k)^\mu \equiv \theta^{\mu\nu}k_\nu = -k_\nu\theta^{\nu\mu} \equiv -(k\theta)^\mu$, $(k\theta p) \equiv k_\mu\theta^{\mu\nu}p_\nu$ and $\theta^{\mu\nu\rho} = \theta^{\mu\nu}\gamma^\rho + \theta^{\nu\rho}\gamma^\mu + \theta^{\rho\mu}\gamma^\nu$.

5.1 Minimal NCSM

The θ corrections to vertices containing quarks are obtained using (10) and (17). The Yukawa part of the action (19) has to be taken into account as well, because it generates additional mass-proportional terms which modify some interaction vertices. In comparison with the SM, this is a novel feature. According to (4), the gauge boson couplings receive θ proportional corrections to three- and four-gluon couplings.

First, let us define the three-gauge boson vertex function

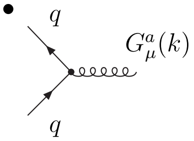
$$
\begin{aligned}
&\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3)) \\
&= -(k_1\theta k_2)\,[(k_1-k_2)^\rho g^{\mu\nu} + (k_2-k_3)^\mu g^{\nu\rho} \\
&\qquad +(k_3-k_1)^\nu g^{\rho\mu}] \\
&\quad - \theta^{\mu\nu}\,[k_1^\rho\,(k_2k_3) - k_2^\rho\,(k_1k_3)] \\
&\quad - \theta^{\nu\rho}\,[k_2^\mu\,(k_3k_1) - k_3^\mu\,(k_2k_1)] \\
&\quad - \theta^{\rho\mu}\,[k_3^\nu\,(k_1k_2) - k_1^\nu\,(k_3k_2)] \\
&\quad + (\theta k_2)^\mu\,\left[g^{\nu\rho}\,k_3^2 - k_3^\nu k_3^\rho\right] + (\theta k_3)^\mu\,\left[g^{\nu\rho}\,k_2^2 - k_2^\nu k_2^\rho\right] \\
&\quad + (\theta k_3)^\nu\,\left[g^{\mu\rho}\,k_1^2 - k_1^\mu k_1^\rho\right] + (\theta k_1)^\nu\,\left[g^{\mu\rho}\,k_3^2 - k_3^\mu k_3^\rho\right] \\
&\quad + (\theta k_1)^\rho\,\left[g^{\mu\nu}\,k_2^2 - k_2^\mu k_2^\nu\right] + (\theta k_2)^\rho\,\left[g^{\mu\nu}\,k_1^2 - k_1^\mu k_1^\nu\right],
\end{aligned} \tag{25}
$$

which is the same function as in [1], but written in the explicit form.

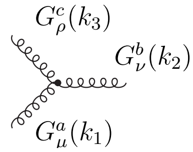
Similarly, the result of the four-gauge boson vertex function is

$$
\begin{aligned}
&\Theta_4((\mu,k_1),(\nu,k_2),(\rho,k_3),(\sigma,k_4)) \\
&= (k_3\theta k_4)\,(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\
&\quad + \theta^{\mu\nu}[k_3^\sigma k_4^\rho - g^{\rho\sigma}(k_3k_4)] + \theta^{\rho\sigma}(k_3^\mu k_4^\nu - k_3^\nu k_4^\mu) \\
&\quad - \theta^{\mu\rho}[k_3^\sigma k_4^\nu - g^{\nu\sigma}(k_3k_4)] - \theta^{\mu\sigma}[k_3^\nu k_4^\rho - g^{\nu\rho}(k_3k_4)] \\
&\quad + \theta^{\nu\rho}[k_3^\sigma k_4^\mu - g^{\mu\sigma}(k_3k_4)] + \theta^{\nu\sigma}[k_3^\mu k_4^\rho - g^{\mu\rho}(k_3k_4)] \\
&\quad + (\theta k_3)^\mu\,(k_4^\nu\,g^{\rho\sigma} - k_4^\rho\,g^{\nu\sigma}) \\
&\quad + (\theta k_4)^\mu\,(k_3^\nu\,g^{\rho\sigma} - k_3^\sigma\,g^{\nu\rho}) \\
&\quad - (\theta k_3)^\nu\,(k_4^\mu\,g^{\rho\sigma} - k_4^\rho\,g^{\mu\sigma}) \\
&\quad - (\theta k_4)^\nu\,(k_3^\mu\,g^{\rho\sigma} - k_3^\sigma\,g^{\mu\rho}) \\
&\quad + (\theta k_3)^\rho\,(k_4^\mu\,g^{\nu\sigma} - k_4^\nu\,g^{\mu\sigma}) \\
&\quad - (\theta k_4)^\rho\,(k_3^\mu\,g^{\nu\sigma} - k_3^\nu\,g^{\mu\rho}) \\
&\quad - (\theta k_3)^\sigma\,(k_4^\mu\,g^{\nu\rho} - k_4^\nu\,g^{\mu\rho}) \\
&\quad + (\theta k_4)^\sigma\,(k_3^\mu\,g^{\nu\rho} - k_3^\nu\,g^{\mu\rho}).
\end{aligned} \tag{26}
$$

The three vertices that appear in the commutative SM as well are

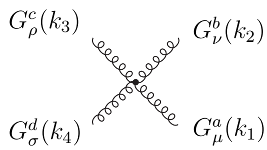
- 

$$
\begin{aligned}
&\mathrm{i}\,g_s\left[\gamma_\mu - \frac{\mathrm{i}}{2}\,k^\nu\,(\theta_{\mu\nu\rho}\,p_{\rm in}^\rho - \theta_{\mu\nu}\,m_q\,)\right]\,T_S^a \\
&\quad = \mathrm{i}\,g_s\,\gamma_\mu\,T_S^a \\
&\quad + \frac{1}{2}\,g_s\,\left[(p_{\rm out}\theta p_{\rm in})\gamma_\mu - (p_{\rm out}\theta)_\mu(\not{p}_{\rm in} - m_q)\right. \\
&\quad \left. - (\not{p}_{\rm out} - m_q)(\theta p_{\rm in})_\mu\right]\,T_S^a,
\end{aligned} \tag{27}
$$

- 

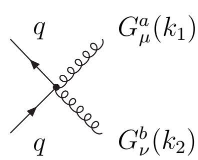
$$
\begin{aligned}
&g_s\,f^{abc}\,[g_{\mu\nu}(k_1-k_2)_\rho + g_{\nu\rho}(k_2-k_3)_\mu \\
&\quad + g_{\rho\mu}(k_3-k_1)_\nu] \\
&\quad + \frac{1}{2}\,g_s\,d^{abc}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3)).
\end{aligned} \tag{28}
$$

The θ-corrected $gggg$ vertex takes the following form:

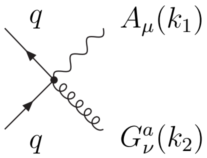
- 

$$
\begin{aligned}
&\mathrm{i}g_s^2\,\{f^{abx}f^{cdx}\,(g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma}) \\
&\quad + f^{acx}f^{bdx}\,(g^{\mu\sigma}g^{\nu\rho} - g^{\mu\nu}g^{\rho\sigma}) \\
&\quad + f^{adx}f^{bcx}\,(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\rho\sigma})\} \\
&\quad + \frac{\mathrm{i}}{2}\,g_s^2\,\{f^{abx}d^{cdx}\Theta_4((\mu,k_1),(\nu,k_2),(\rho,k_3),(\sigma,k_4)) \\
&\quad + [(\mu,k_1,a) \leftrightarrow (\rho,k_3,c)] \\
&\quad + [(\mu,k_1,a) \leftrightarrow (\sigma,k_4,d)] \\
&\quad + [(\nu,k_2,b) \leftrightarrow (\rho,k_3,c)] + [(\nu,k_2,b) \leftrightarrow (\sigma,k_4,d)] \\
&\quad + [(\mu,k_1,a) \leftrightarrow (\rho,k_3,c),(\nu,k_2,b) \leftrightarrow (\sigma,k_4,d)]\}.
\end{aligned} \tag{29}
$$

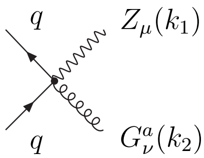
Equations (10) and (17) describe the interaction vertices involving quarks and two or three gauge bosons. These do not appear in the SM. In the following we provide all contributions to such vertices with four legs and the corresponding mass-proportional contributions:

- 

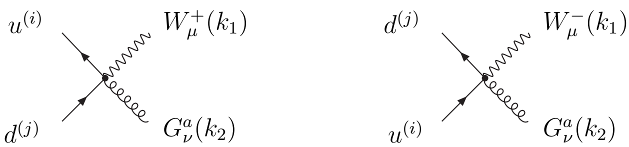
$$-\frac{g_s^2}{2}\,\{\theta_{\mu\nu\rho}\,(k_1^\rho-k_2^\rho)\,T_S^aT_S^b + \mathrm{i}\,[\theta_{\mu\nu\rho}\,(p_{\mathrm{in}}^\rho+k_2^\rho)-\theta_{\mu\nu}m_q]\,f^{abc}T_S^c\}\,, \qquad (30)$$

- 

$$-\frac{1}{2}\,g_s\,e\,Q_q\,\theta_{\mu\nu\rho}\,(k_1^\rho-k_2^\rho)\,T_S^a, \qquad (31)$$

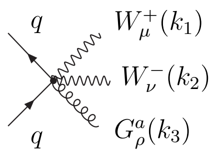
- 

$$\frac{-\,e\,g_s}{2\sin 2\theta_{\mathrm{W}}}\Big[\theta_{\mu\nu\rho}\,(k_1^\rho-k_2^\rho)(c_{V,q}-c_{A,q}\gamma_5) +2\theta_{\mu\nu}\,m_q\,c_{A,q}\gamma_5\Big]\,T_S^a, \qquad (32)$$

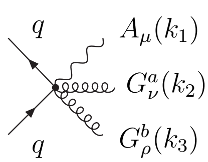
- 

$$\frac{-e\,g_s}{4\sqrt{2}\sin\theta_{\mathrm{W}}}\begin{pmatrix}V_{ij}\\ V_{ij}^*\end{pmatrix}\Big\{\,\theta_{\mu\nu\rho}(k_1^\rho-k_2^\rho)\,(1-\gamma_5) \qquad (33)$$
$$-\,\theta_{\mu\nu}\left[\begin{pmatrix}m_{u^{(i)}}\\ m_{d^{(j)}}\end{pmatrix}(1-\gamma_5)-\begin{pmatrix}m_{d^{(j)}}\\ m_{u^{(i)}}\end{pmatrix}(1+\gamma_5)\right]\Big\}\,T_S^a.$$

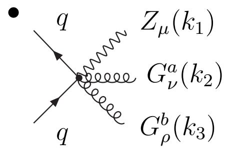
Similarly, the five-field vertices $qqWWg$, $qq\gamma gg$, $qqZgg$, $qqW\gamma g$, $qqWZg$ and $qqWgg$ are extracted from (10) and (17) as well. They have no mass-proportional corrections:

- 

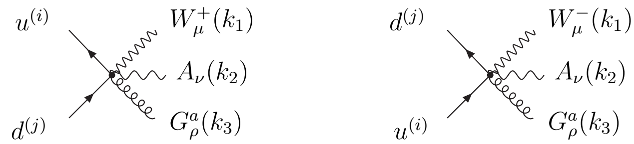
$$-\frac{e^2\,g_s}{8\sin^2\theta_{\mathrm{W}}}\theta_{\mu\nu\rho}\,(1-\gamma_5)\,T_S^a, \qquad (34)$$

- 

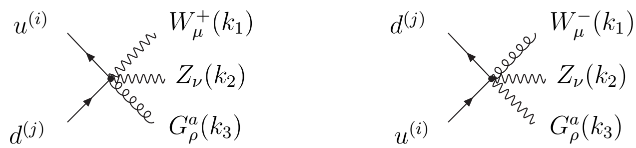
$$-\frac{\mathrm{i}}{2}e\,g_s^2\,Q_q\theta_{\mu\nu\rho}\,f^{abc}T_S^c, \qquad (35)$$

- 

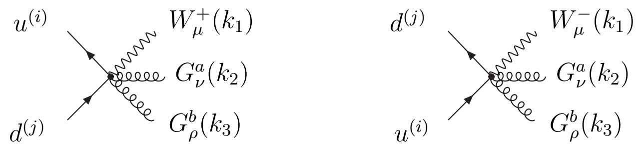
$$-\frac{\mathrm{i}\,e\,g_s^2}{2\sin 2\theta_{\mathrm{W}}}\theta_{\mu\nu\rho}\,(c_{V,q}-c_{A,q}\gamma_5)\,f^{abc}T_S^c, \qquad (36)$$

- 

$$\frac{e^2\,g_s}{4\sqrt{2}\sin\theta_{\mathrm{W}}}\begin{pmatrix}V_{ij}\\ -V_{ij}^*\end{pmatrix}\theta_{\mu\nu\rho}\,(1-\gamma_5)\,T_S^a, \qquad (37)$$

- 

$$\frac{e^2\,g_s\,\cos\theta_{\mathrm{W}}}{4\sqrt{2}\sin^2\theta_{\mathrm{W}}}\begin{pmatrix}V_{ij}\\ -V_{ij}^*\end{pmatrix}\theta_{\mu\nu\rho}\,(1-\gamma_5)\,T_S^a, \qquad (38)$$

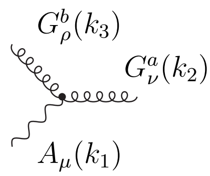
- 

$$\frac{-\mathrm{i}\,e\,g_s^2}{4\sqrt{2}\sin\theta_{\mathrm{W}}}\begin{pmatrix}V_{ij}\\ V_{ij}^*\end{pmatrix}\theta_{\mu\nu\rho}\,(1-\gamma_5)\;f^{abc}\,T_S^c. \qquad (39)$$

The Feynman rules for pure QCD vertices, (27)–(30), have already been given in [13].

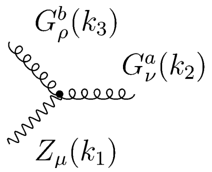
5.2 Non-minimal NCSM

Here we give the Feynman rules for θ corrections to the gauge boson vertices Zgg and γgg in the non-minimal NCSM introduced in [16]. Observe that the quark sector is not affected by the change of the representation in the gauge part of the action, and, consequently, is the same in both models, the mNCSM and the nmNCSM one.

- 

$$-2\,e\,\sin 2\theta_{\mathrm{W}}\mathrm{K}_{\gamma gg}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,\delta^{ab}, \qquad (40)$$

•



$$-2\, e\, \sin 2\theta_{\rm W} {\rm K}_{Zgg}\, \Theta_3((\mu, k_1), (\nu, k_2), (\rho, k_3))\, \delta^{ab}. \quad (41)$$

The constants K are not independent, and they are defined in (7), from where ${\rm K}_{Zgg} = -\tan\theta_{\rm W}{\rm K}_{\gamma gg}$.

6 Conclusions

This article, together with [1], represents a complete description of the non-commutative standard model constructed in [2, 16] and makes it accessible to further research.

We have presented a careful discussion of QCD-electroweak charged and neutral currents as well as a detailed analysis of the Yukawa part of the NCSM action. The NCSM action is expressed in terms of physical fields and mass eigenstates, up to the first order in the non-commutative parameter θ.

The novel feature of the NCSM presented in this article is the presence of mixtures of gluon interactions with electroweak ones. We again encounter the mass-proportional corrections to the quark–boson couplings, stemming from the Yukawa part of the action (19). These features are introduced by the Seiberg–Witten maps.

With these interactions provided, together with [1], we complete the analysis of all interactions of the NCSM appearing at the order θ, and hope that this will enable further investigations leading to stringent bounds to the non-commutativity parameters.

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